

22087406



**MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2**

Thursday 8 May 2008 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 19]

Give all answers in this question correct to the **nearest dollar**.

Clara wants to buy some land. She can choose between two different payment options. Both options require her to pay for the land in **20** monthly installments.

Option 1: The first installment is \$2500. Each installment is \$200 more than the one before.

Option 2: The first installment is \$2000. Each installment is 8% more than the one before.

(a) If Clara chooses option 1,

(i) write down the values of the second and third installments;

(ii) calculate the value of the final installment;

(iii) show that the **total amount** that Clara would pay for the land is \$88 000. [7 marks]

(b) If Clara chooses option 2,

(i) find the value of the second installment;

(ii) show that the value of the fifth installment is \$2721. [4 marks]

(c) The price of the land is \$80 000. In option 1 her total repayments are \$88 000 over the **20** months. Find the annual rate of simple interest which gives this total. [4 marks]

(d) Clara knows that the **total amount** she would pay for the land is not the same for both options. She wants to spend the least amount of money. Find how much she will save by choosing the cheaper option. [4 marks]

2. [Maximum mark: 22]

- (i) A survey of 400 people is carried out by a market research organization in two different cities, Buenos Aires and Montevideo. The people are asked which brand of cereal they prefer out of Chocos, Zucos or Fruti. The table below summarizes their responses.

	Chocos	Zucos	Fruti	Total
Buenos Aires	43	85	62	190
Montevideo	57	35	118	210
Total	100	120	180	400

- (a) One person is chosen at random from those surveyed. Find the probability that this person
- (i) does not prefer Zucos;
- (ii) prefers Chocos, given that they live in Montevideo. [4 marks]
- (b) Two people are chosen at random from those surveyed. Find the probability that they both prefer Fruti. [3 marks]

The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-squared test at the 5% level of significance is performed.

- (c) State the null hypothesis. [1 mark]
- (d) State the number of degrees of freedom. [1 mark]
- (e) Show that the expected frequency for the number of people who live in Montevideo and prefer Zucos is 63. [2 marks]
- (f) Write down the chi-squared statistic for this data. [2 marks]
- (g) State whether the market research organization would accept the null hypothesis. Clearly justify your answer. [2 marks]

(This question continues on the following page)

(Question 2 continued)

- (ii) The following table shows the cost in AUD of seven paperback books chosen at random, together with the number of pages in each book.

Book	1	2	3	4	5	6	7
Number of pages (x)	50	120	200	330	400	450	630
Cost (y AUD)	6.00	5.40	7.20	4.60	7.60	5.80	5.20

- (a) Plot these pairs of values on a scatter diagram. Use a scale of 1 cm to represent 50 pages on the horizontal axis and 1 cm to represent 1 AUD on the vertical axis. *[3 marks]*
- (b) Write down the linear correlation coefficient, r , for the data. *[2 marks]*
- (c) Stephen wishes to buy a paperback book which has 350 pages in it. He plans to draw a line of best fit to determine the price. State whether or not this is an appropriate method in this case and justify your answer. *[2 marks]*

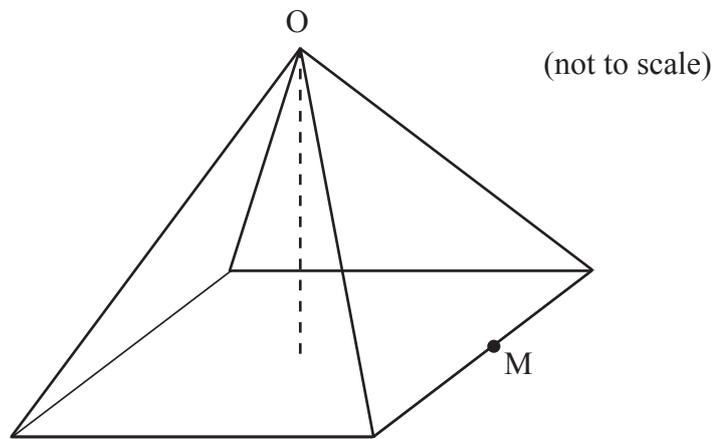
3. [Maximum mark: 14]

(i) Amir needs to construct an isosceles triangle ABC whose area is 100 cm^2 . The equal sides, AB and BC, are 20 cm long.

(a) Angle ABC is acute. Show that the angle ABC is 30° . [2 marks]

(b) Find the length of AC. [3 marks]

(ii) Sylvia is making a square-based pyramid. Each triangle has a base of length 12 cm and a height of 10 cm.



(a) Show that the **height** of the pyramid is 8 cm. [2 marks]

M is the midpoint of the base of one of the triangles and O is the apex of the pyramid.

(b) Find the angle that the line MO makes with the base of the pyramid. [3 marks]

(c) Calculate the volume of the pyramid. [2 marks]

(d) Daniel wants to make a rectangular prism with the same volume as that of Sylvia's pyramid. The base of his prism is to be a square of side 10 cm. Calculate the height of the prism. [2 marks]

4. [Maximum mark: 23]

(i) Consider the function $f : x \mapsto \frac{kx}{2^x}$.

(a) Given that $f(1) = 2$, show that $k = 4$. [2 marks]

(b) Write down the values of q and r for the following table.

x	-1	0	1	2	4	8
$f(x)$	-8	0	2	q	1	r

[2 marks]

As x increases from -1, the graph of $y = f(x)$ reaches a maximum value and then decreases, behaving asymptotically.

(c) Draw the graph of $y = f(x)$ for $-1 \leq x \leq 8$. Use a scale of 1 cm to represent 1 unit on both axes. The position of the maximum, M, the y -intercept and the asymptotic behaviour should be clearly shown. [4 marks]

(d) Using your graphic display calculator, find the coordinates of M, the maximum point on the graph of $y = f(x)$. [2 marks]

(e) Write down the equation of the horizontal asymptote to the graph of $y = f(x)$. [2 marks]

(f) (i) Draw and label the line $y = 1$ on your graph.

(ii) The equation $f(x) = 1$ has two solutions. One of the solutions is $x = 4$. Use your **graph** to find the other solution. [4 marks]

(ii) The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x}$$

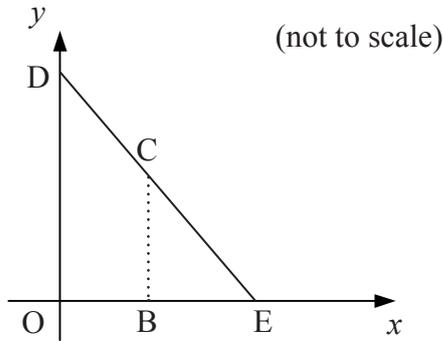
(a) Find $C'(x)$. [3 marks]

(b) Show that the cost per person is a minimum when 10 people are invited to the party. [2 marks]

(c) Calculate the minimum cost per person. [2 marks]

5. [Maximum mark: 12]

On the coordinate axes below, D is a point on the y -axis and E is a point on the x -axis. O is the origin. The equation of the line DE is $y + \frac{1}{2}x = 4$.



(a) Write down the coordinates of point E. [2 marks]

C is a point on the line DE. B is a point on the x -axis such that BC is parallel to the y -axis. The x -coordinate of C is t .

(b) Show that the y -coordinate of C is $4 - \frac{1}{2}t$. [2 marks]

OBCD is a trapezium. The y -coordinate of point D is 4.

(c) Show that the area of OBCD is $4t - \frac{1}{4}t^2$. [3 marks]

(d) The area of OBCD is 9.75 square units. Write down a quadratic equation that expresses this information. [1 mark]

(e) (i) Using your graphic display calculator, or otherwise, find the two solutions to the quadratic equation written in part (d).

(ii) Hence find the correct value for t . Give a reason for your answer. [4 marks]